

Isocurvature Constraints and Gravitational Ward Identity

Hojin Yoo

Physics Department, University of Wisconsin-Madison, Madison, WI 53706

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Axions and WIMPZILLAs are well-motivated dark matter candidates with interesting cosmological implications, such as isocurvature perturbations and non-Gaussianities. However, these predicted implications in the literature are based on the assumption that the cross-correlation between curvature and CDM isocurvature is negligible. This contribution discusses the cross-correlation in the axion and the WIMPZILLA scenarios. Particularly, it is shown that the gravitational Ward identity associated with diffeomorphism invariance plays an important role in this cross-correlation calculation confirming the assumption.

1 Introduction

Currently, the cosmological observational data provides strong support for the simplest single field slow-roll inflationary models predicting nearly scale-invariant, adiabatic and Gaussian density perturbations. However, there still exist many inflationary models compatible with the current data. These models may have interesting deviations from the prediction of the simplest inflationary models. In particular, the axion and the WIMPZILLA scenarios [1, 2] in inflationary models can give rise to CDM isocurvature perturbation and non-Gaussianities, which are detectable by ongoing and near future experiments.

In the literature, the cosmological implication of these scenarios has been studied with the assumption that the cross-correlation between curvature perturbation and isocurvature perturbation is negligible. The careful prediction of the cross-correlation is very important because the current observational constraints of the CDM isocurvature significantly depend on the cross-correlation. However, the validity of the assumption is not evident because of the gravitational interaction of perturbations.

In this contribution, we discuss the curvature and isocurvature cross-correlation $\langle \zeta \delta_S \rangle$ for the axion and the WIMPZILLA scenarios. Particularly, we emphasize the importance of diffeomorphism invariance in the computation of the cross-correlation. We present the result of the computation, which shows that the cross-correlation is too small to be detectable through the current CMB experiments, and this provides proof of the validity of the assumption.

2 Observational constraints on isocurvature perturbations

The current observational data shows that the CMB power spectrum is consistent with the adiabatic initial condition. However, it does not completely rule out contributions from CDM

isocurvature, but it provides the constraints on the primordial isocurvature perturbations, which is usually parameterized by two variables

$$\alpha \equiv \frac{\Delta_{\delta_S}^2}{\Delta_\zeta^2 + \Delta_{\delta_S}^2}, \quad \beta \equiv \frac{\Delta_{\zeta\delta_S}^2}{\sqrt{\Delta_\zeta^2 \Delta_{\delta_S}^2}},$$

where Δ_ζ^2 , $\Delta_{\delta_S}^2$, and $\Delta_{\zeta\delta_S}^2$ are the power spectra of adiabatic, isocurvature perturbations, and their cross-correlation, respectively, at the primordial epoch. The isocurvature contribution to the CMB temperature perturbation is expected to be roughly less than 10% compared to the curvature contribution. More precisely, based on the WMAP+BAO+ H_0 data [3],

$$\alpha_0 < 0.077 \text{ (95\% CL)}, \text{ and } \alpha_{-1} < 0.0047 \text{ (95\% CL)},$$

where the subscript denotes the parameter β , which is the fractional cross-correlation. The significant difference in the upper-bound of α between uncorrelated and totally (anti-)correlated cases originates from the different behaviors of the adiabatic and isocurvature radiation transfer functions. In particular, the transfer function of isocurvature perturbations on small scales is suppressed by an additional factor k_{eq}/k compared to that of adiabatic perturbations. Therefore, when the cross-correlation is not negligible $\Delta_{\zeta\delta_S}^2 \sim \sqrt{\Delta_\zeta^2 \Delta_{\delta_S}^2}$, the cross-correlation contribution $\Delta_{\zeta\delta_S}^2$ to the CMB temperature perturbation is generally larger than the pure isocurvature contribution $\Delta_{\delta_S}^2$ on small scales. This explains why the upper limit of α for totally (anti-)correlated models is tighter than that for uncorrelated models.

Therefore, estimating the cross-correlation is crucial to give correct predictions and restrict the parameters of isocurvature models. For instance, the axion scenario with a negligible homogeneous vacuum misalignment angle (and similarly the WIMPZILLA scenario with a negligible homogenous background field value) predicts detectable non-Gaussianity [2, 4]

$$f_{NL} \sim 30 \left(\frac{\alpha}{0.067} \right)^{3/2}$$

if the assumption that the cross-correlation is zero or negligible is valid. In the following section, we discuss the cross-correlation for the axion and the WIMPZILLA scenarios.

3 Cross-correlation for axion and WIMPZILLA scenarios

We consider the axion and the WIMPZILLA scenarios discussed in Refs. [2, 5]. Particularly, we assume that Peccei-Quinn symmetry is spontaneously broken during inflation for axions. In both scenarios, when the spatial average value of the axion or the WIMPZILLA is much less than its inhomogeneity, the isocurvature perturbation generated from their density perturbations in the comoving gauge is written as

$$\delta_S = \omega \frac{\sigma^2 - \langle \sigma^2 \rangle}{\langle \sigma^2 \rangle},$$

where σ denotes an axion or WIMPZILLA field, and $\omega \equiv \Omega_\sigma / \Omega_{CDM}$. For axions, $\sigma = f_a \delta\theta_a$, where f_a is the PQ symmetry breaking scale and $\delta\theta_a$ is the inhomogeneity of the phase of axion.

In order to obtain the cross-correlation $\langle \zeta \delta_S \rangle$, we calculate the correlator $\langle \zeta \sigma^2 \rangle$ using “in-in” formalism [6]. We only consider the gravitational coupling whose interaction Hamiltonian is

derived from the ADM formalism with a gauge choice. With the comoving gauge ($\delta\phi = 0$), the cubic interaction Hamiltonian is

$$H_{\zeta\sigma\sigma}^I(t) = -\frac{1}{2} \int d^3x a^3(t) T_{\sigma}^{\mu\nu}(t, \vec{x}) \delta g_{\mu\nu}(t, \vec{x}),$$

$$\delta g_{\mu\nu} = \begin{pmatrix} -2\frac{\dot{\zeta}}{H} & \partial_i \left(-\frac{\zeta}{H} + \epsilon \frac{a^2}{\nabla^2} \dot{\zeta} \right) \\ \partial_i \left(-\frac{\zeta}{H} + \epsilon \frac{a^2}{\nabla^2} \dot{\zeta} \right) & a^2 \delta_{ij} 2\zeta \end{pmatrix},$$

where $T_{\sigma}^{\mu\nu}$ is the stress energy tensor of the field σ , and $\delta g_{\mu\nu}$ is the scalar metric perturbation. Then the two-point correlator is written as

$$\widetilde{\langle \zeta \sigma^2 \rangle}_p = \int d^3x e^{-i\vec{p}\cdot\vec{x}} \int^t d^4z a^3(t_z) \left\langle \left[\zeta(t, \vec{x}) \sigma^2(t, 0), \frac{i}{2} (2\zeta a^2 \delta_{ij} T_{\sigma}^{ij})_z \right] \right\rangle + \mathcal{O}\left(\frac{p^2}{a^2}\right), \quad (1)$$

where the contributions from other interaction terms are $\mathcal{O}(p^2/a^2)$ because they are derivatively coupled with ζ . One might try to estimate the integral using the super-horizon approximation frequently used to compute correlators in the inflationary de Sitter background, which yields a large cross-correlation. However, the estimation fails because of the explicit breaking of diffeomorphism invariance.

In fact, the integral (1) turns out to be small because of the Ward identity associated with diffeomorphism invariance, especially the spatial dilatation, i.e., $x^\mu \rightarrow \tilde{x}^\mu + \lambda X^\mu$, where $X^\mu(x) = (0, x^1, x^2, x^3)$. Notice that the curvature perturbation ζ is smoothly extended to the spatial dilatation transform by taking the external momentum \vec{p} to zero [7]. This fact with the Ward identity allows us to factorize $\langle \zeta \zeta \rangle$ from a correlator involving external ζ in the $p \rightarrow 0$ limit. The well-known example is the consistency relation for the 3-point function of ζ in the squeezed limit [8]

$$\langle \zeta_{\vec{p}_1} \zeta_{\vec{p}_2} \zeta_{\vec{p}_3} \rangle \xrightarrow{p_1 \rightarrow 0} -(2\pi)^3 \delta^3(\sum_i \vec{p}_i) |\zeta_{p_1}^o|^2 |\zeta_{p_2}^o|^2 \frac{\partial}{\partial \ln p_2} \ln p_2^3 |\zeta_{p_2}^o|^2,$$

where $\langle \zeta_{\vec{p}_1} \zeta_{\vec{p}_2} \rangle = (2\pi)^3 \delta^3(\vec{p}_1 + \vec{p}_2) |\zeta_{p_1}^o|^2$. Similarly, in the two-point function case, we find

$$\widetilde{\langle \zeta \sigma^2 \rangle}_p \xrightarrow{p \rightarrow 0} |\zeta_p^o|^2 \frac{\partial}{\partial \ln a} \langle \sigma^2 \rangle.$$

Note that here we assume that UV divergences are properly treated. See Ref. [9] for details.

In our recent work [9], this computation has been done explicitly, and we have found that

$$\widetilde{\langle \zeta \sigma^2 \rangle}_p = |\zeta_p^o|^2 \times \begin{cases} \frac{\Gamma^2(\nu) H^2}{H_p^2 \pi^3} \left(\frac{p}{2aH}\right)^{3-2\nu} & \text{for scalar in de Sitter space} \\ \frac{H_p^2}{4\pi^2} & \text{for massless scalar} \end{cases} + \mathcal{O}\left(\frac{p^2}{a^2}\right), \quad (2)$$

where H_p denotes the Hubble scale at which scale p exits the horizon, $\nu \equiv \sqrt{9/4 - m_\sigma^2/H^2}$ and $p^3 |\zeta_p^o|^2 = 2\pi^2 \Delta_\zeta^2 = H_p^2/4M_p^2\epsilon$. In addition, in the dS space-time, we have

$$\widetilde{\langle \sigma^2 \sigma^2 \rangle}_p = \frac{1}{2\pi^2} \frac{H^4}{p^3} \frac{1}{3-2\nu} \left(\frac{p}{aH}\right)^{6-4\nu} \left[1 - \left(\frac{\Lambda_{IR}}{p}\right)^{3-2\nu} \right], \quad (3)$$

where $m_\sigma < 3H/2$. Combining the results (2) and (3) yields the fractional cross-correlation $\beta = \langle \zeta \sigma^2 \rangle / \sqrt{\langle \sigma^2 \sigma^2 \rangle} |\zeta_p^o|^2 \lesssim \frac{\Delta_\zeta}{2}$, which is quite small; however, this result is still interesting because the cross-correlation is not decaying, and because it is independent of the parameters, such as m_σ and ω .

4 Discussion and Conclusion

The above result shows that the fractional cross-correlation $\beta \lesssim \Delta_\zeta/2 \approx 2.5 \times 10^{-5}$ in both scenarios, which is too small to be measured through the current CMB experiment. More precisely, in order to have approximately the same level of the CMB power spectrums from pure isocurvature and cross-correlation at the intermediate scale $l \sim 200$, i.e. $C_l^{iso} \sim C_l^{cor}$, the fractional cross-correlation should be at least $\beta \sim 4 \times 10^{-2}$.

The smallness of the cross-correlation is understood by the fact that the super-horizon mode of the curvature perturbation ζ can be smoothly extended to the gauge mode, which is the spatial dilatation in the $p \rightarrow 0$ limit. Physically, the curvature perturbation ζ can affect the particle density ρ_σ and generate small cross-correlation only at its horizon crossing, since ζ freezes after horizon exit and effectively becomes a gauge mode. Thus this verifies the assumption that the cross-correlation is negligible in the axion and the WIMPZILLA scenarios.

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